



Deep
Generative
Models for
Creating
Synthetic
Microstructures

Viraj Shah, Balaji Sesha Sarath Pokuri, Rahul Singh,
Chinmay Hegde, Baskar Ganapathysubramanian, Soumik Sarkar

Contact email: viraj@iastate.edu

Objective

- A key problem in computational material science deals with understanding the effect of material distribution (i.e., **microstructure**) on material performance.
- The challenge we consider here is to **synthesize microstructures with desired physical and chemical properties**, given a finite number of microstructure images, evaluated based on the **physical invariances that the microstructure exhibits**.
- Conventional approaches are based on stochastic optimization and are **computationally intensive**.
- We introduce Machine learning based generative models for the **fast synthesis of binary microstructure** images.
- Our model is a **Wasserstein Generative Adversarial Network** that uses a finite number of training images to **synthesize new microstructures that satisfy the physical invariances** respected by the original data.
- For the training of our model, we curated a dataset of **Binary 2D microstructural images of polymer phase separation**. We made our dataset available publically.

- In this presentation, we discuss:
 1. Details of **dataset generation**
 2. **Physical invariances** under consideration: volume fraction and two-point correlation
 3. Introduction to deep generative adversarial networks (**GANs**) and our method.
 4. **Microstructure images generated** by our WGAN model, and their analysis
 5. **Conclusion** on usefulness and effectiveness of our model

1.1 Dataset

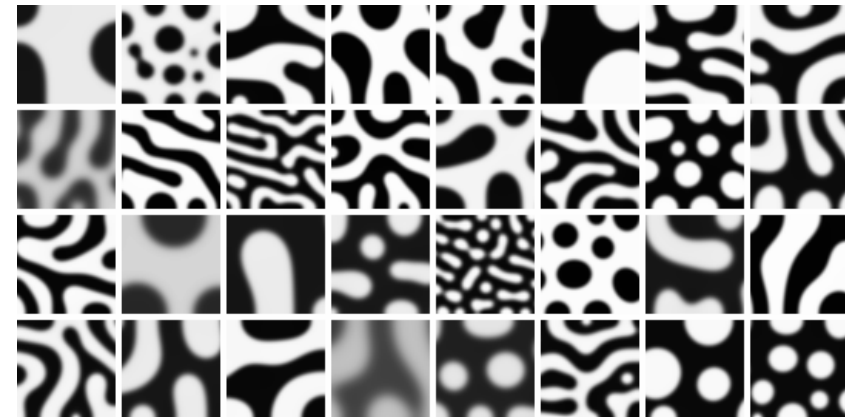
We curated a dataset of **Binary 2D** microstructural images of polymer phase separation (CH - dataset).

- This dataset was generated through the simulation of a time evolving Cahn -Hilliard equation [1], describing **phase separation in binary polymer blends**. Thus, we call it CH-dataset.
- Several realizations of the equation were done through **different values of volume fractions and binary interaction parameters**.
- Morphologies were outputted at **constant** time intervals.

Details of the dataset

- Collection of **34672 grayscale images**,
- resolution of each image is **101x101**.
- Each pixel takes a floating point value **between 0 and 1**.
- Dataset and supporting code is made public at:
<https://zenodo.org/record/2580293>

Samples from the dataset



1.2 Dataset Generation

dataset was generated through the simulation of a time evolving Cahn -Hilliard equation [1]:

$$\frac{\partial \phi}{\partial t} = \nabla \left(M \nabla \left(\frac{\partial f}{\partial \phi} - \epsilon^2 \nabla^2 \phi \right) \right)$$
$$f(\phi) = \phi \ln(\phi) + (1 - \phi) \ln(1 - \phi) + \chi_{12} \phi(1 - \phi)$$

- Simulations were performed on a **101×101** (pixels) square domain, for **10 values** of $\chi_{12} \in [2.2, 3.4]$;
 10 values of total volume fraction $\phi_0 = \int_{\Omega} \phi d\Omega \in [0.3, 0.5]$. Also, $\epsilon = 1e - 2$, $l_x, l_y = 1$, $M = 1$, $dt = 1e - 3$.
 - Data was saved after every prescribed number of **time steps (20 time steps)**.
 - Data was then **augmented** by reflecting and flipping the phases(black/white)
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- Because data was generated using an **energy dissipating equation**, data generated using one simulation has:
 - The same (volume) fraction for all times ($\int_{\Omega} \phi d\Omega = \text{constant} \forall t$),
 - With increasing time, the average domain size increases as $r_{avg} \propto t^{\frac{1}{3}}$. i.e., r_{avg} increases with time.

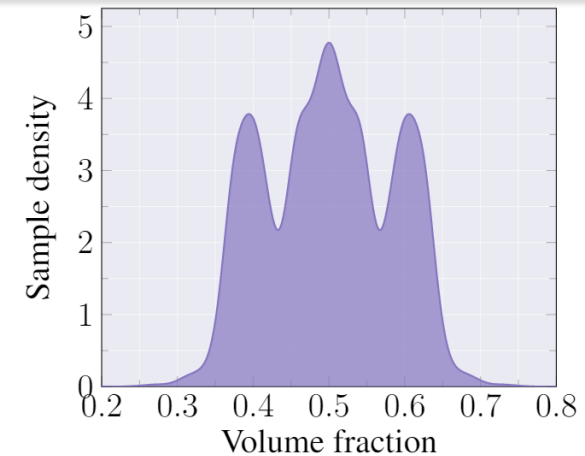
2. Physical Invariances

We consider the underlying material to be a **two-phase homogeneous, isotropic** material.
Our setup for statistical characterization of microstructure follows with [2].

- A phase function $\phi^{(1)}(.)$ is used to characterize this two-phase system, defined as: $\phi^{(1)}(\mathbf{r}) = \begin{cases} 1, \mathbf{r} \in V_1, \\ 0, \mathbf{r} \in V_2, \end{cases}$
where V_1 is the region occupied by phase 1 and V_2 is the region occupied by phase 2.
- The 1-point correlation function, $p_1^{(1)}$, commonly known as **volume fraction**, is defined as: $p_1^{(1)} = \mathbb{E}_{\mathbf{r}} \phi^{(1)}(\mathbf{r})$
- The **two-point correlation function**, $p_2^{(1)}$, is defined as: $p_2^{(1)}(r_{12}) = \mathbb{E}_{\mathbf{r}_1, \mathbf{r}_2} [\phi^{(1)}(\mathbf{r}_1) \phi^{(1)}(\mathbf{r}_2)]$

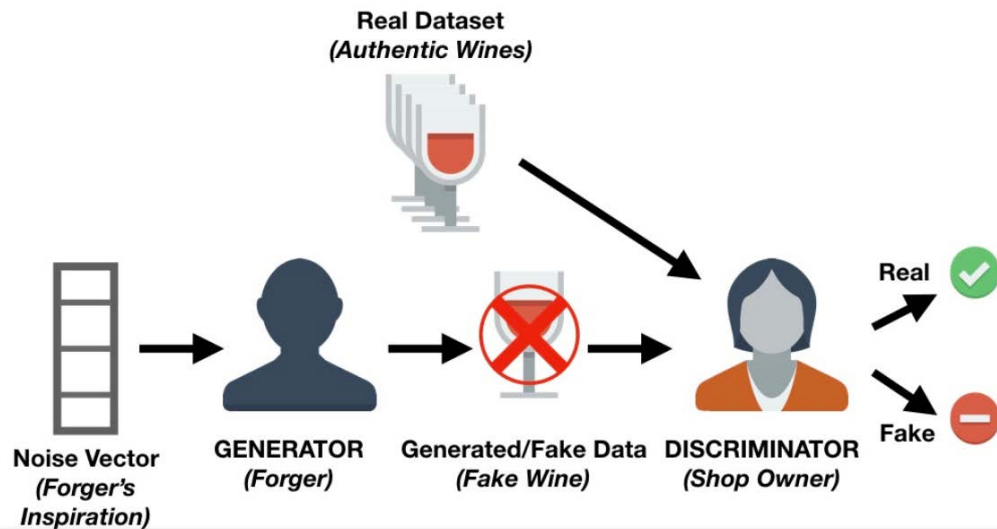
- Our aim is to synthesize microstructures that satisfy certain **target statistical properties** of the material distribution.
- We choose these statistical properties to be **total volume fraction of a material (p_1)** and **2-point correlation (p_2)**.

Distribution of volume fraction for our dataset



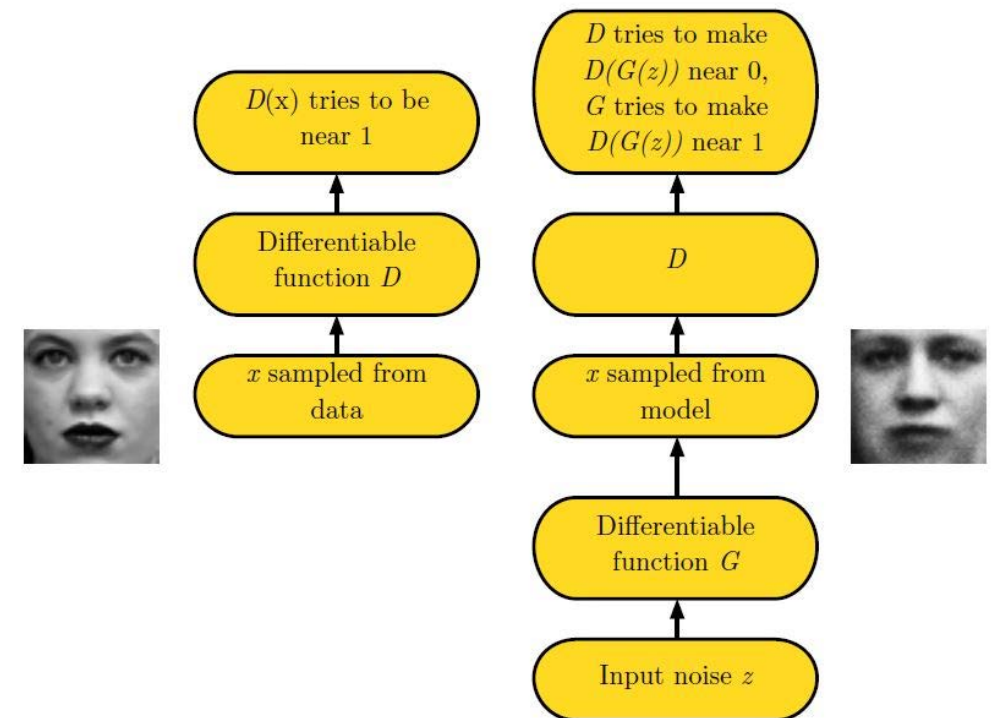
3.1 Introduction to Generative Adversarial Networks [4]

The basic idea of GANs is to set up a minimax game between two players: **generator** and **discriminator** [4].



- We can think of the **generator** as being like a forger, trying to make fake wine, and the **discriminator** as being like wine-shop owner, trying to allow legitimate wines and catch fake wines.
- To succeed in this game, the **forger** must learn to make wine that is **indistinguishable from genuine wine** implying that the generator network must learn to create samples that are drawn from the same distribution as the training data.

GAN model

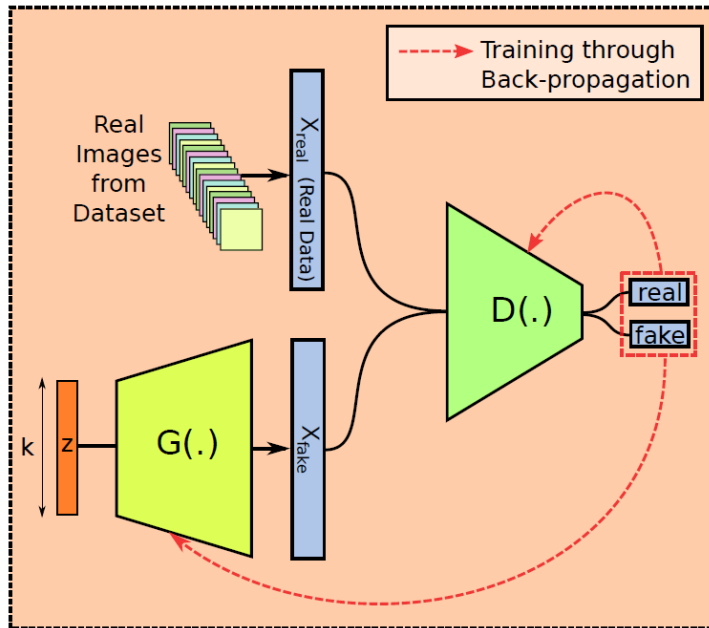


$$\min_G \max_D V(D, G) = E_{x \sim p_{data}(x)} [\log D(x)] + E_{z \sim p_z(z)} [\log(1 - D(G(z)))]$$

Image credits: (left) datacamp blog, (right) GAN tutorial by Ian Goodfellow

3.2 Our methodology

We use our CH dataset as the training data for a improved version of GAN – called Wasserstein GAN with Gradient Penalty [5].



WGAN-GP-CH

- Trained over CH dataset that contains all 34672 images covering entire range of p_1 and p_2 .

WGAN-GP-CHp1

- Trained over CHp1 dataset with ~6000 images of CH dataset having a volume fraction (p_1) value between 0.35 to 0.45.

WGAN-GP-CHp2

- Trained over CHp2 dataset with ~3000 images of CH dataset having a (p_2) value equal to 0.0625.

- Using the Cahn-Hilliard (CH) dataset, we prepare two smaller datasets referred as CHp1 and CHp2 by segregating the images based on their p_1 and p_2 values respectively.
- The first subset CHp1 is a collection of all images of CH dataset having a volume fraction (p_1) value between 0.35 to 0.45.
- The second subset is segregated on the basis of p_2 values of images, and contain all the images from CH dataset having 2 – point correlation (p_2) value equal to 0.0625.
- We train 3 WGAN-GP using CH, CHp1 and CHp2 as the training data respectively.
- As these segregated datasets typically contain images with similar statistical properties (either p_1 or p_2), we can testify the ability of our model to preserve such properties by observing the p_1 or p_2 values of the images generated by these 3 networks.

4.1 Results

Our trained WGAN-GP models are able to generate microstructures closely resembling the real microstructures. Moreover, these generated microstructures respect the statistical constraints (p_1 and p_2)

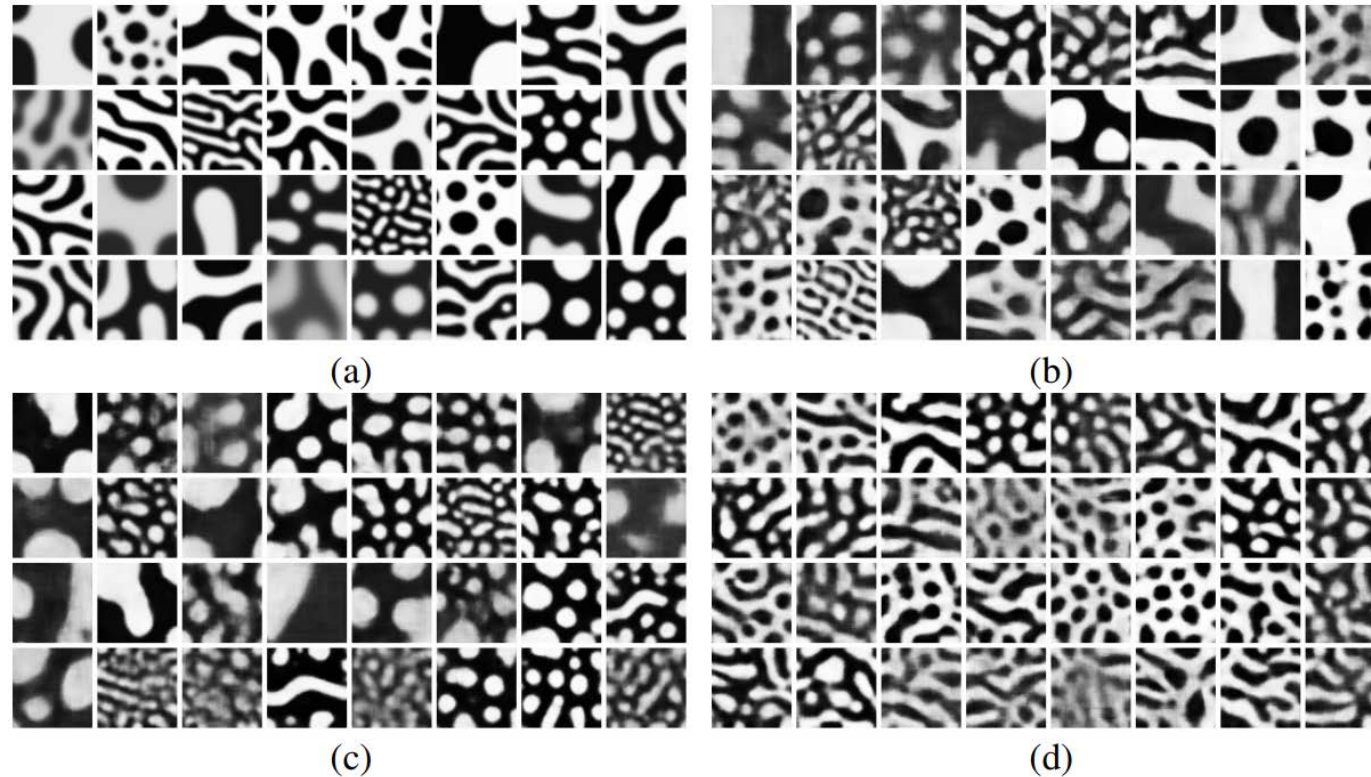


Figure 2: (a) Sample images from Cahn-Hilliard dataset; (b) samples generated by WGAN-GP trained on CH-dataset; (c) Samples generated by WGAN-GP trained over the morphologies from CH_{p1} dataset (only includes the images with volume fraction between 0.35 to 0.45); (d) Samples generated by WGAN-GP trained over the morphologies from CH_{p2} dataset (only includes the images with 2-point correlation equal to 0.0625).

4.2 Analysis of the Results

We analyze (i) the **statistical properties** of the generated microstructures; (ii) **interpolation behavior** of our trained WGAN models; (iii) **Free energy of generated microstructures** to present interesting findings.

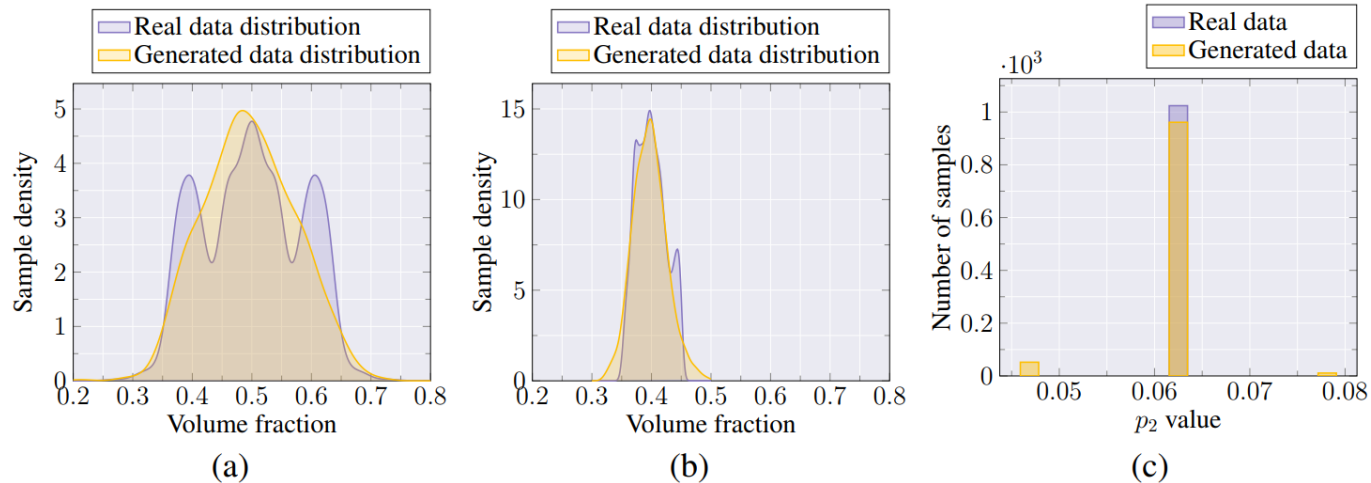
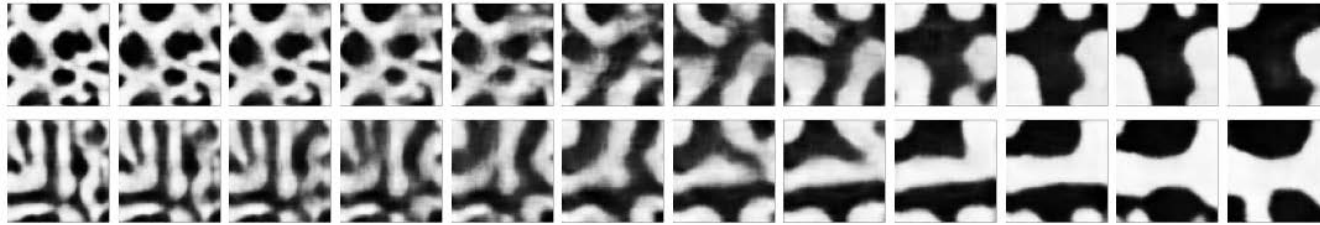


Figure 3: (a) Comparisons of the distributions of volume fractions of training dataset and that of the samples generated by WGAN-GP trained over entire CH dataset; (b) Comparisons of the distributions of volume fractions of training dataset and that of the samples generated by WGAN-GP trained over the CH_{p1} dataset; (c) Histograms of p_2 correlation values for samples from CH_{p2} dataset and samples generated by WGAN-GP trained over CH_{p2} dataset.

- Fig. 3 provides the density plots/histograms of p_1 and p_2 values of the images for both training data and generated data.
- The striking similarities in the spread of both the density plots/histograms suggest that our **network successfully reproduces the statistical properties** of the real (training) images in the simulated images.
- In Fig. 3 (a,b), densities of p_1 value is compared between the real data and generated data for network trained on CH dataset and CH_{p1} dataset.
- We provide the histogram for p_2 values for models trained using CH_{p2} in Fig. 3 (c). **Both the histograms closely match.**

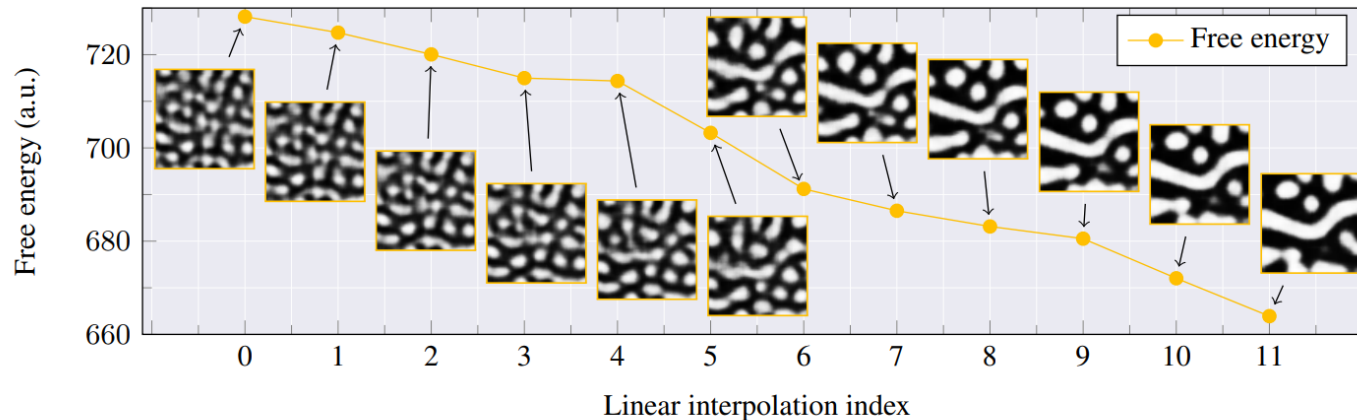
4.3 Analysis of the Results

We display interesting behavior of the learned image manifold through results of interpolation over latent vectors z



- Randomly pick two different noise vectors z_1 and z_2 , and linearly interpolate between them to obtain 10 more such noise vectors. All 12 images are plotted as in Fig. 4.

Figure 4: (a) Results of the linear interpolation over latent variable z for WGAN-GP trained over the entire CH dataset



(b)

Figure 5: (a) Results of the linear interpolation over latent variable z for WGAN-GP trained over the images from CH_{p1} dataset (only includes the images with volume fraction between 0.35 to 0.45); volume fraction values are printed above each image; (b) Free energy of the interpolated morphologies (volume fraction of each image is $(0.41 \pm 5\%)$).

- Very interestingly, we observe that an unseen invariance of energy minimization is captured in the interpolation. As shown in Fig. 5 (b), the free energy of the morphologies decreases as the we move from one interpolation step to the next.
- In this process, the volume fraction (p_1) is also **preserved within reasonable limits**. This suggests that the WGAN framework is able to learn latent physical rules from the dataset.
- Such behavior can effectively be used to decide **series of manufacturing processes** for obtaining final morphology from initial morphology **without adding any new material**.

5. Conclusion

- We curated a **Binary 2D microstructural images of polymer phase separation (CH - dataset)** by solving CH equations and successfully used it with a machine learning model that **approaches computational results in microstructure synthesis tasks**.
- We also made our dataset **publically available**.
- We train three different WGAN models with full or subset of our CH dataset. We show that the generated images **respect the distribution of certain physical invariances** - specifically, can be used for synthesizing a promising new material that meets a desired performance target.
- We analyze our results to come up with **interesting properties of our trained WGAN models** such as interpolation and free energy reduction behavior and explore previously unknown correlations in **process–structure–property linkages**.

References

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4. I. Goodfellow, J. Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative adversarial nets. In *NeurIPS*, 2014.
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