RECONSTRUCTION FROM PERIODIC NONLINEARITIES, WITH APPLICATIONS TO HDR IMAGING

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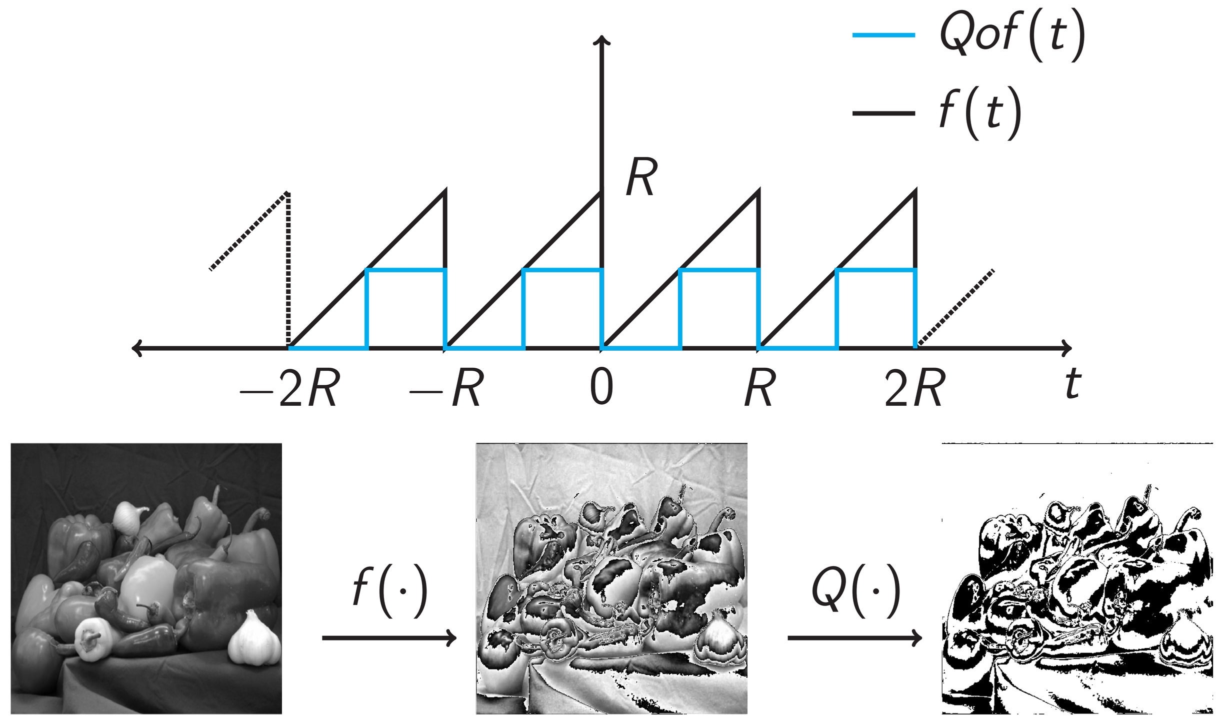
Problem Setup

- **Aim**: Reliable estimation of a signal from **periodic nonlinearities**.
- Nonlinearity in each observation is well-modeled by a periodic function such as a sinusoidal function, or sawtooth function, or square-wave function.
- We focus on a periodic nonlinear observation model encountered in high-dynamic range **(HDR) imaging.**

HDR Imaging and Modulo camera [1]

Intensity camera   Modulo camera   Recovered image [1]

Modulo operation and Quantization

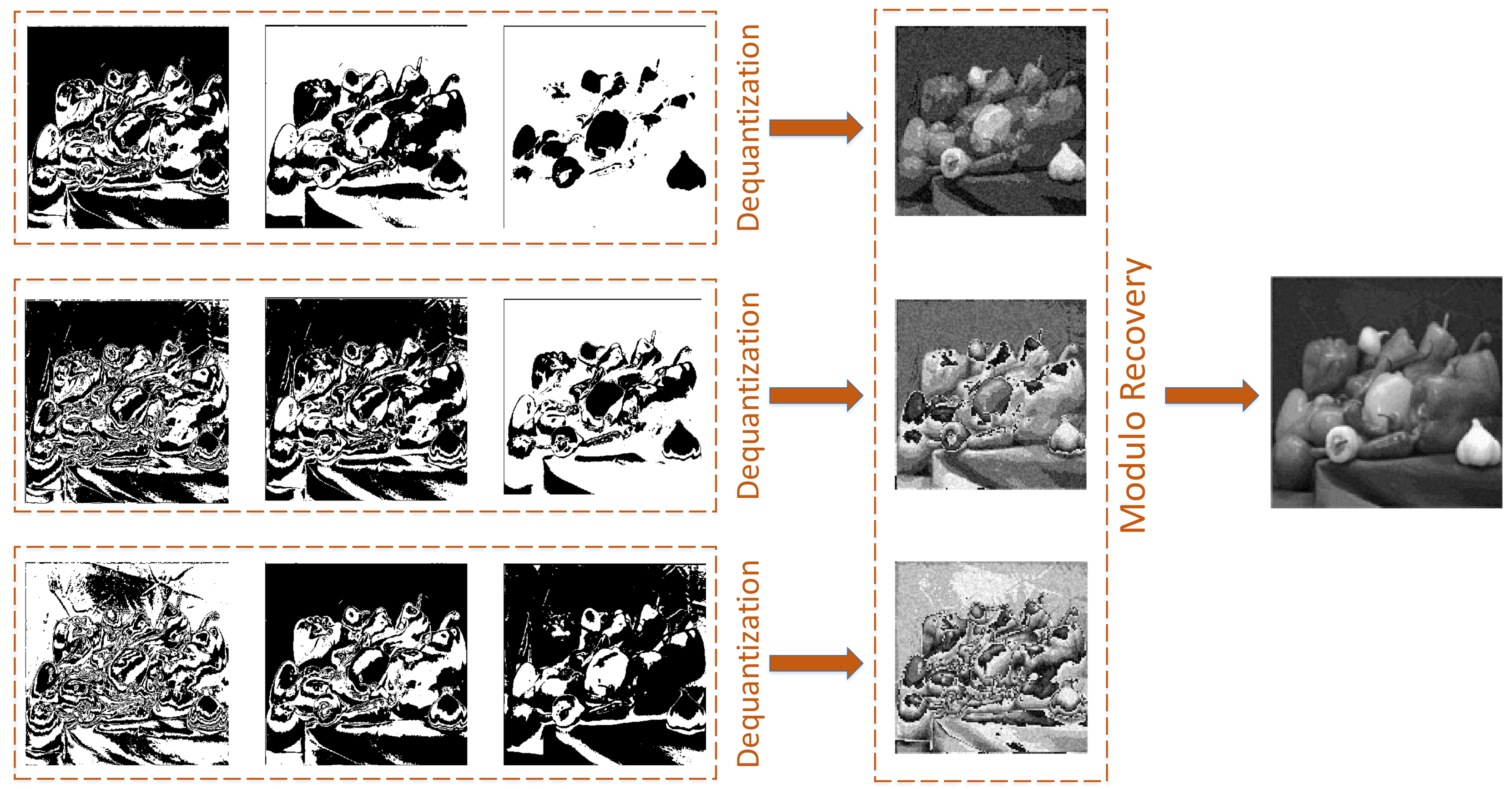


Mathematical Model

$$y = Q\left(\begin{bmatrix} C^0 \\ C^1 \\ \vdots \\ C^{k-1} \end{bmatrix} \bmod \left(\begin{bmatrix} D^1 \\ D^2 \\ \vdots \\ D^{k'} \end{bmatrix} Bx\right)\right).$$

- $B \in \mathbb{R}^{q\times n}$ - Sparse basis matrix satisfying RIP condition.
- $D \in \mathbb{R}^{p\times n}$ - Block diagonal matrix with k' blocks that contains
  - uniformly distributed random variables in RQM algorithm.
  - elements chosen in geometric progression for RQM-multi-shot.
- $C \in \mathbb{R}^{m\times p}$ - Block diagonal matrix with k blocks and contains multipliers chosen in harmonic progression.

Our Algorithm: RQM-Recovery from Quantized Modulo images



Stagewise approach

- **Dequantization**: Recover u from $y = Q(Cu)$ using HM algorithm.
- **Modulo recovery**: Recover z from $u = f(Dx)$ using MF-Sparse [2] or Multi-shot UHDR [1].
- **Sparse recovery**: Recover x from $z = Bx$ using any stable sparse recovery algorithm (e.g. CoSaMP).

Algorithm:RQM

Inputs: $y, D, B, C, k, k', \Omega, s$
Output: $\widehat{x}$
Stage 1: Harmonic dequantization
$\widehat{u} \leftarrow \text{HMDequantization}(y, C, k)$
Stage 2: Modulo recovery[2]
$\theta \leftarrow \exp(i\widehat{u})$
for $l = 1 : q$ **do**
  $t \leftarrow D(l : q : (k'-1)q + l, l)$
  $\phi \leftarrow \theta(l : q : (k'-1)q + l)$
  $\widehat{z}_l = \text{argmax}_{\omega\in\Omega} |\langle y, \psi_\omega\rangle|$
end for
$\widehat{z} \leftarrow [\widehat{z}_1, \widehat{z}_2 \dots, \widehat{z}_q]^T$
Stage 3: Sparse recovery
$\widehat{x} \leftarrow \text{CoSaMP}(\widehat{z}, B, s)$

HM Algorithm: Dequantization using Harmonic Multipliers

For every element u_i, we measure $y_{i,j} = Q(c_{i,j}u_i)$. with $c_{i,0} = 1$, and each subsequent $c_{i,j}$ is defined as :

$$c_{i,j} = \begin{cases} \frac{k}{k-j}, & \text{if } y_{i,0} = 0, \\ \frac{k}{k+j}, & \text{if } y_{i,0} = 1, \end{cases} \quad j = 1, 2, \dots, k-1.$$

The underlying idea is to increase or decrease the value of $c_{i,j}u_i$ gradually and to detect the index j^* for which $y_{i,j}$ changes its value for the first time.

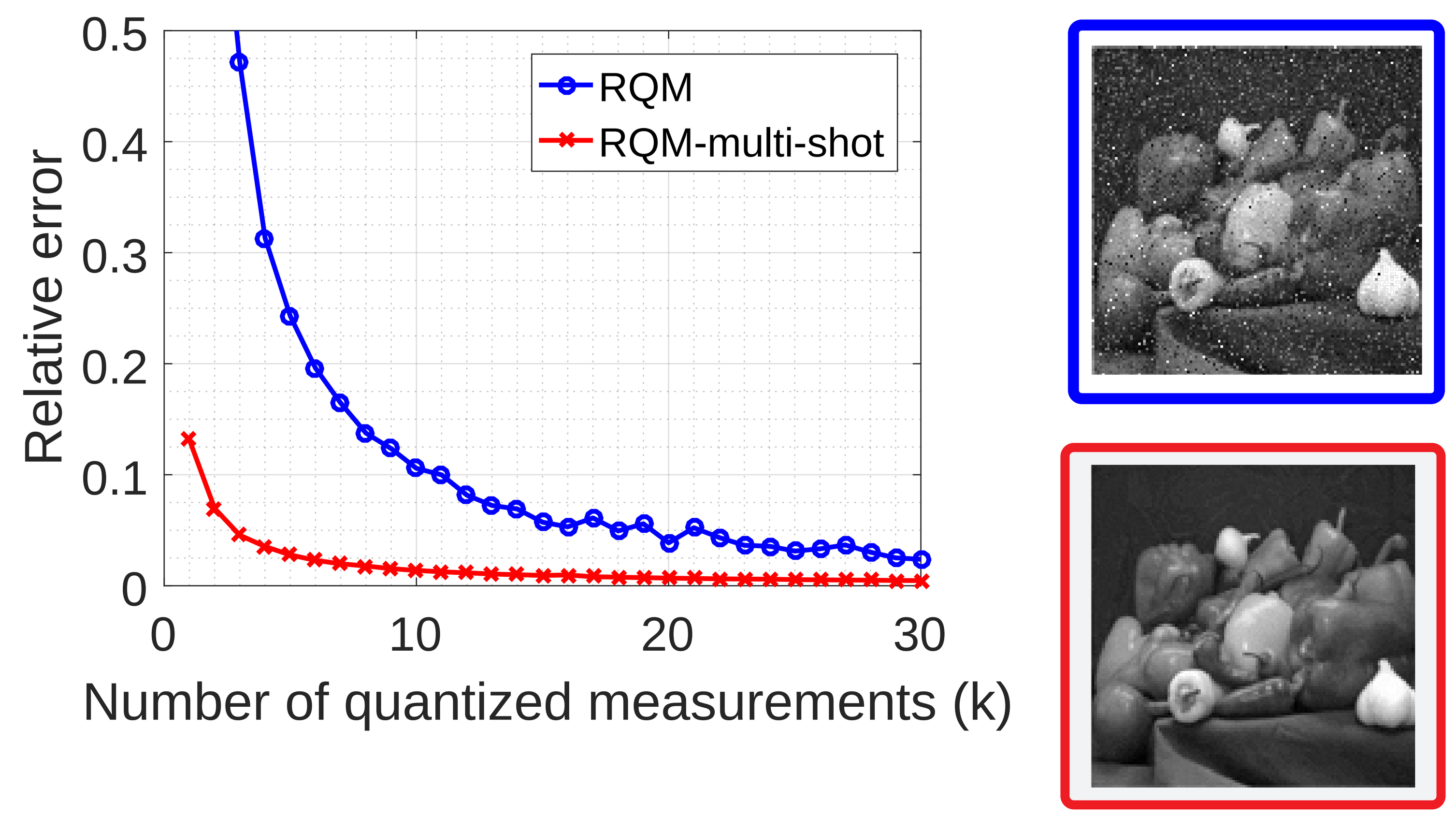
if $y_l = 0$ **then**
  $t \leftarrow y(l+n : n : (k-1)n+l, 1)$
  $j^* \leftarrow \min_{j\in\{1,2,\dots,k-1\}}$ s.t. $t_j = 1$
  $\widehat{u}_l \leftarrow v \sim U[\Delta\frac{k-j^*}{k}, \Delta\frac{k-j^*+1}{k}]$
end if

if $y_l = 1$ **then**
  $t \leftarrow y(l+n : n : (k-1)n+l, 1)$
  $j^* \leftarrow \min_{j\in\{1,2,\dots,k-1\}}$ s.t. $t_j = 0$
  $\widehat{u}_l \leftarrow v \sim U[\Delta\frac{k+j^*-1}{k}, \Delta\frac{k+j^*}{k}]$
end if

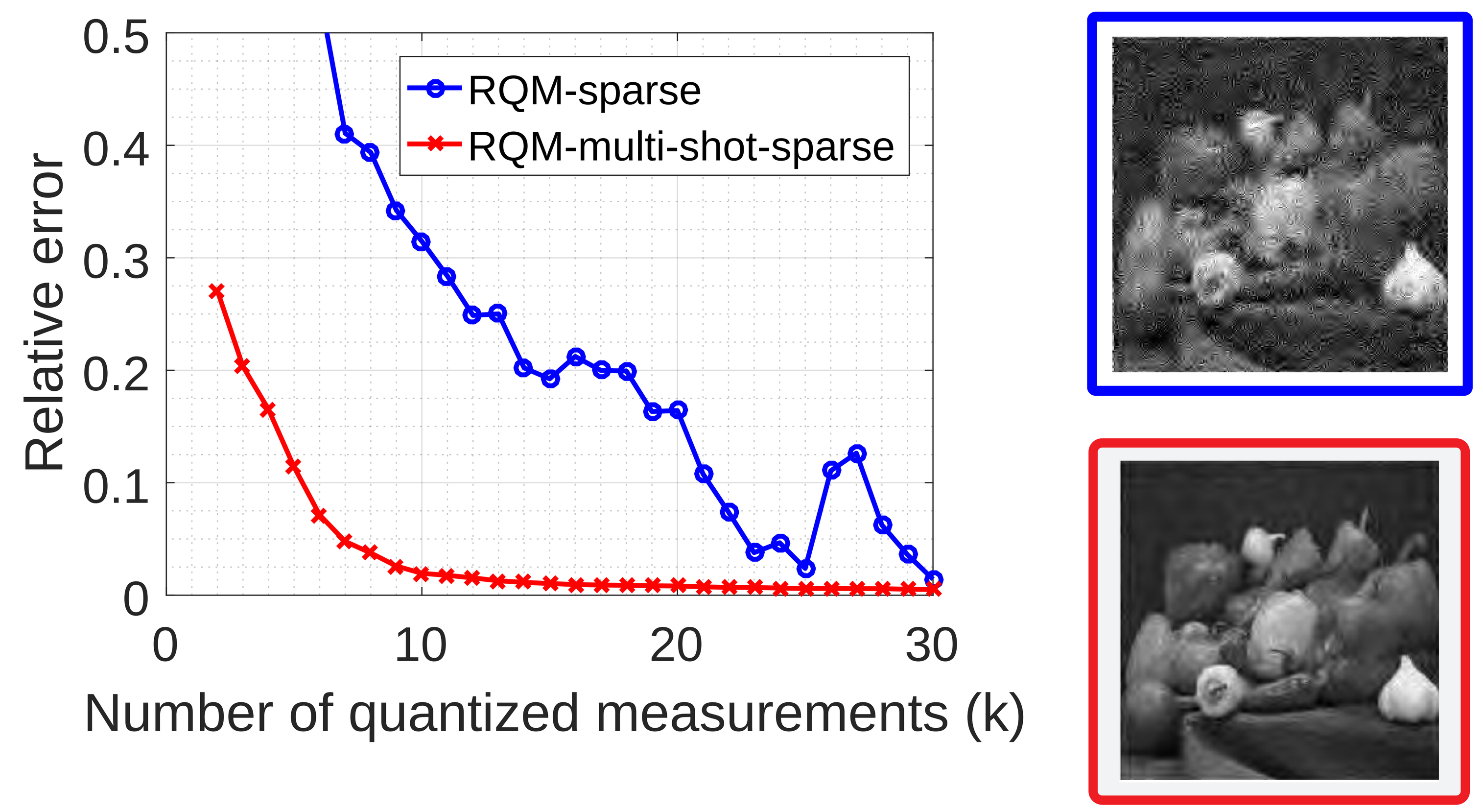
Experimental Results

Recovery from Quantized Modulo images

without sparsity



with sparsity



Acknowledgments

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References

[1] H. Zhao, B. Shi, C. Fernandez-Cull, S. Yeung, and R. Raskar."Unbounded High Dynamic Range Photography using a Modulo Camera." Intl. Conf. on Comp. Photography (ICCP), 2015.
[2] M. Soltani and C. Hegde, "Stable recovery from random sinusoidal feature maps." IEEE Int. Conf. Acoust., Speech, and Signal Processing (ICASSP), 2017